

The problem of an efficiency increase of an FEL amplifier is now of great practical importance. Technique of undulator tapering in the post-saturation regime is used at the existing x-ray FELs LCLS and SACLA, and is planned for use at the European XFEL, Swiss FEL, and PAL XFEL. There are also discussions on the future of high peak and average power FELs for scientific and industrial applications. In this paper we perform detailed analysis of the tapering strategies for high power seeded FEL amplifiers. Application of similarity techniques allows us to derive universal law of the undulator tapering.

Principle of undulator tapering

Equations, describing motion of the particles in the ponderomotive potential well of electromagnetic wave and undulator get simple form when written down in normalized form:

$$\frac{d\psi}{dz} = \dot{C} + \dot{P}, \quad \frac{d\dot{P}}{dz} = U \cos(\phi_U + \psi),$$

where $\dot{z} = \Gamma z$, and U and ϕ_U are amplitude and phase of effective potential. Energy change of electrons is small in the exponential stage of amplification, $\dot{P} \ll 1$, and process of electron bunching in phase ψ lasts for long distance, $\dot{z} \gg 1$. Situation changes drastically when electron energy change \dot{P} approaches to the unity. The change of phase on the scale of $\Delta\dot{z} \approx 1$ becomes to be fast, particles start to slip in phase ψ which leads to the debunching of the electron beam modulation, and growth of the radiation power is saturated. Undulator tapering, i.e. adjustment of the detuning according to the energy loss of electrons, $\dot{C}(\dot{z}) = -\dot{P}(\dot{z})$, allows to keep synchronism of electrons with electromagnetic wave and increase output power.

Similarity techniques

Main physical parameters of the problem:

$B = 2\Gamma\sigma^2\omega/c$ - diffraction parameter

$\dot{C} = C/\Gamma$ - detuning parameter

Other parameters:

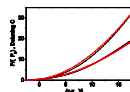
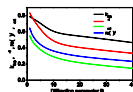
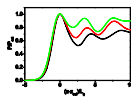
$C = 2\pi/\lambda_w - \omega/(2c\gamma^2)$ - the detuning

$\Gamma = [L_w^2\theta_0^2 A_0^2/(4\lambda_w^2\gamma^2)]^{1/2}$ - the gain parameter

$\dot{P} = (E - E_0)/(\rho E_0)$ - energy deviation

$\rho = c\gamma^2/\omega$ - the efficiency parameter

Field gain re (A/Γ) and efficiency in the saturation, $\eta_{sat} = P_{sat}/(P_0\lambda)$ of the FEL amplifier tuned to exact resonance are defined by the only diffraction parameter B . Operation of the FEL amplifier before saturation is also defined by the diffraction parameter B . When amplification process enters nonlinear stage, output power is function of two parameters, diffraction parameter and reduced undulator length.



Universal characteristics of FEL amplifier. Color codes are: black - trapping efficiency K_{trap} for globally optimized undulator; red - fitting coefficient of global optimization α_{opt} ; blue - FEL field gain $\text{Re}(A/\Gamma)$; green - FEL efficiency in the saturation, $\eta_{sat} = P_{sat}/(P_0\lambda)$.

Phase space distribution of electrons (top) and population of electrons in energy at different stages of trapping process. Color codes correspond to different location of the particles in the beam (black - core of the beam, blue - edge of the beam). Here diffraction parameter is $B = 10$. Top, middle, and bottom plots correspond to $(z - z_{sat})/L_g = 2.5, 3.9$ and 5.3 , respectively.

Particles in the core of bunch (black points) are trapped most effectively. Nearly all particles located in the edge of the electron beam (blue points) leave stability region very soon. Trapping process lasts for several field gain length when trapped particles become to be isolated in the trapped energy band for which undulator tapering is optimized further. For specific value of the diffraction parameter $B = 10$ it is not finished even at three field gain lengths after saturation. Non-trapped particles continue to populate low energy tail of the energy distribution forming energy bands after excursion of a path in the phase space governed by nonlinear dynamics. This effect has been observed experimentally at LCLS.

Universal law of undulator tapering

FEL radiation is coherent radiation of the electron beam which is modulated at the resonance wavelength during amplification process. It is reasonable here to remember properties of the radiation of the modulated electron beam. Radiation power of modulated beam in helical undulator is given by:

$$P = \frac{\pi\theta_0^2\omega_0^2 a_{00}^2}{4\pi c^2} z \left[\arctan\left(\frac{1}{2N}\right) + N \ln\left(\frac{4N^2}{4N^2 + 1}\right) \right],$$

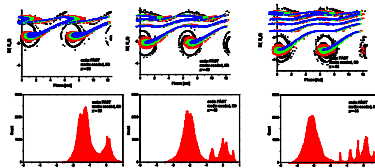
where a_{00} is amplitude of modulation of the electron beam current ($I(z, t) = I_0[1 + a_{00}\cos\omega(z/v_z - t)]$), and $N = k\sigma^2/z$ is Fresnel number.

This expression is a crucial element for understanding the optimum law of the undulator tapering. Indeed, in the deep tapering regime some fraction of the particles is trapped in the regime of coherent deceleration. Thus, beam modulation is fixed, and asymptotically radiation power should be described by this expression. One can easily find that asymptotes of undulator tapering, 1D model of (wide electron beam), and thin beam asymptote are well described by this expression. Asymptote of wide electron beam corresponds to large values of Fresnel number N , and radiation power scales as $P \propto z^2$. Asymptote of thin electron beam corresponds to small values of the Fresnel Number N , and radiation power becomes linearly proportional to the undulator length, $P \propto z$. Undulator tapering should adjust detuning according to the energy loss by electrons, and we find that tapering law should be quadratic for the case of wide electron beam, $C \propto -P \propto z^2$, and linear - for the case of thin electron beam, $C \propto -P \propto z$.

Main essence of our study is to apply parametrical dependence for the modulation of the electron beam to fit optimum detuning pattern such that condition of optimum tapering is preserved:

$$\dot{C} = \alpha_{opt}(\dot{z} - \dot{z}_0) \left[\arctan\left(\frac{1}{2N}\right) + N \ln\left(\frac{4N^2}{4N^2 + 1}\right) \right],$$

with Fresnel number N fitted by $N = \beta_{trap}/(\dot{z} - \dot{z}_0)$. Start of the undulator tapering \dot{z}_0 is firmly fixed by the global optimization procedure, $\dot{z}_0 = z_{sat} - 2L_g$. β_{trap} is rather well approximated with the linear dependency on diffraction parameter, $\beta_{trap} = 10 \times B$. Parameter α_{opt} is plotted in the Figure. It is slow function of the diffraction parameter B , and scales approximately to $B^{1/3}$ as all other important FEL parameters. Accuracy of this fit is pretty good giving the results for optimum detuning which are close to the global optimum.



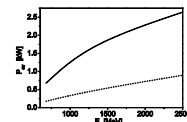
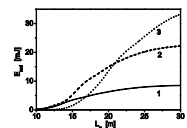
FLASH technology:

Scaling of burst mode to high average power NGL source

13.5 nm

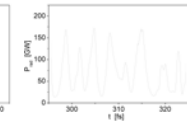
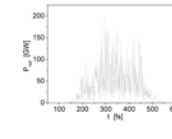
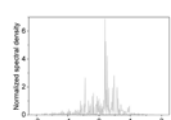
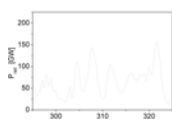
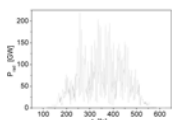
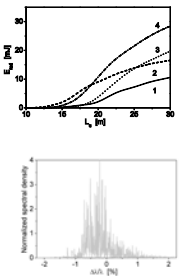
E.A. Schneidmiller, V.F. Vogel, H. Weise and M.V. Yurkov, *Journal of Micro/Nanolithography, MEMS, and MOEMS* 11(2), 021102 (2012).

	FLASH	NGL-680	NGL-1250	NGL-2500
Electron energy, MeV	680	680	1250	2500
Bunch charge, nC	1	1	1	1
Peak current, A	2500	2500	2500	2500
Normalized emittance, mm-mrad	1.5	1.5	1.5	1.5
rms energy spread, MeV	0.5	0.5	0.5	0.5
Macropulse duration, ms	0.8	0.8	0.8	0.8
Macropulse rep. rate, MHz	9	10	10	10
# pulses in macropulse	7200	8000	8000	8000
Macropulse rep. rate, Hz	10	10	10	10
Undulator period, cm	2.73	2.73	3.7	5.0
Undulator length, m	27	30	30	30
Energy in the radiation pulse, mJ	1.4	8.5	22	33
Peak power, GW	5.6	34	88	130
Average radiation power, W	100	680	1760	2640



6.8 nm

	1250	1250	2500	2500
Electron energy, MeV	1	1	1	1
Bunch charge, nC	2500	2500	2500	2500
Peak current, A	1.5	1	1.5	1
Normalized emittance, mm-mrad	0.5	0.5	0.5	0.5
rms energy spread, MeV	0.8	0.8	0.8	0.8
Macropulse duration, ms	10	10	10	10
Macropulse rep. rate, MHz	8000	8000	8000	8000
# pulses in macropulse	10	10	10	10
Macropulse rep. rate, Hz	3.7	3.7	5.0	5.0
Undulator period, cm	30	30	30	30
Undulator length, m	11	16	20	28
Energy in the radiation pulse, mJ	44	64	80	110
Peak power, GW	880	1280	1600	2240
Average radiation power, W				



Scaling of cw mode to high average power NGL source

Summary

- Optimum undulator tapering allows to reach high FEL efficiency, theoretically above 50%.
- This is valid for physical parameters when slippage is less than electron bunch length and coherence length.
- Efficiency of tapered SASE FEL is limited by poor longitudinal coherence.
- Thus, development of schemes with better longitudinal coherence will allow to increase FEL efficiency and increase radiation power.
- A bunch of such schemes is studied theoretically. Experimental verification and scaling of the results to high power devices will take long time.

For the present time we can state:

- SRF accelerator technology and SASE FELs are developed at DESY for 20 years in the framework of TESLA/FLASH/XFEL/ILC projects.
- Both, burst and cw options reached mature status. 17.5 GeV linac for the European XFEL is being built using burst technology. CW option developed at DESY will form base for construction of 4 GeV cw linac at LCLS-II.
- All elements of the accelerators are produced by industry. An experience stored during construction of the European XFEL provides solid base for estimation of the cost of future industrial high power accelerators.
- The physics of SASE FEL is well understood, and experimental results are in good agreement with theoretical predictions. However, undulator tapering still requires more experimental experience. Relevant studies are planned to be performed at FLASH2.
- Both, burst and cw options allows to construct high average power (multi-kW) FEL as a source for the next generation lithography.

ERL scheme similar to that of the year 2000 can be used. Key features of the proposal:

- CW injector.
- Superconducting CW energy recovery linac (1 GeV, 10 mA average current).
- Self-amplified spontaneous emission (SASE) FEL as radiation source (ELV, 10 kW average power).
- Application of the undulator tapering will allow to reduce average beam current, or to increase output power.



C. Papadimitrakis et al., *Nuclear Instruments and Methods in Physics Research A* 642 (2011) 9-27

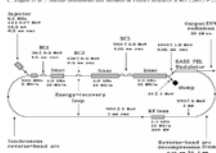


Fig. 2. Basic scheme of the high-power SASE FEL.